

STEP Examiners' Report 2012

Mathematics STEP 9465/9470/9475

November 2012



General Remarks

There were just over 1000 entries for paper II this year, almost exactly the same number as last year. Overall, the paper was found marginally easier than its predecessor, which means that it was pitched at exactly the level intended and produced the hoped-for outcomes. Almost 50 candidates scored 100 marks or more, with more than 400 gaining at least half marks on the paper. At the lower end of the scale, around a quarter of the entry failed to score more than 40 marks. It was pleasing to note that the advice of recent years, encouraging students not to make attempts at lots of early parts to questions but rather to spend their time getting to grips with the six that can count towards their paper total, was more obviously being heeded in 2012 than I can recall being the case previously.

As in previous years, the pure maths questions provided the bulk of candidates' work, with relatively few efforts to be found at the applied ones. Questions 1 and 2 were the most popular questions, although each drew only around 800 "hits" – fewer than usual. Questions 3 - 5 & 8 were almost as popular (around 700), with Q6 attracting the interest of under 450 candidates and Q7 under 200. Q9 was the most popular applied question – and, as it turned out, the most successfully attempted question on the paper – with very little interest shown in the rest of Sections B or C.

Comments on individual questions

Q1 The first question is set with the intention that everyone should be able to attempt it, but 20% of candidates were clearly put off by the algebraic nature of this year's opener. It was, nonetheless, the most popular question on the paper, possibly due to the lack of any advanced techniques. The specifically numerical parts of the question were generally more confidently, and hence successfully, handled than the general ones. So, for instance, formulae for the various coefficients in (i) were often not correct, even when high marks were scored on the question. When numerical answers went astray, it was usually due to incorrect signs in the early stages, with candidates failing to realise that all terms were positive. The very final demand (for the coefficient of x^{66}) was the real test, not only of candidates' resilience and nerve but also of their grasp of where the various contributions were coming from. From a marking point of view, very few candidates gave particularly clear methods, and it was usually difficult for the markers to decipher the underlying processes from what appeared to be merely a whole load of numbers written down and added up.

Q2 This turned out to be the second most popular question and the highest scoring of the pure questions. Explanations apart, most candidates held their nerve remarkably well to produce careful algebra leading to correct answers. The added "trap" in part (i) – in that each answer contained an arbitrary constant – caught many out.

Q3 This was another popular question, scoring just over half of the marks on average. Although most efforts to establish the given initial result were eventually successful, many made hard work of it, failing to notice the obvious result that if $t = \sqrt{x^2 + 1} + x$ then $\frac{1}{t} = \sqrt{x^2 + 1} - x$. The first integral could then be found by realising that $f(t) = \frac{1}{t^2}$ was the relevant function here,

or by repeating the substitution already used.

Quite a few candidates thought that the second integral followed from the first, which was unfortunate, as it didn't. However, most efforts at this second integral were unsuccessful anyhow, with candidates usually getting 3 of the 10 marks for setting up the substitution and then often going round in circles. The main problem lay in using sin and cos instead of tan and sec, or

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in continuing with $\sqrt{x^2 + 1} + x$ without identifying a suitable function f(t). It was helpful to find this, but not essential.

Q4 This was quite a popular question, as candidates seemed to like using the log series, and appreciated the helpful structuring of the question. However, inequalities are seldom entirely confidently handled, and explanations (wherever they are required) are generally rather feeble. Thus, several marks were often not picked up, sometimes because the candidates did not think they had to consider addressing issues such as whether the series was valid in this case. Part (i) was usually fairly well done, with (ii) providing more of a challenge. Part (iii) required only informal arguments, but many scored only 1 of the 2 marks allocated here due to being a bit too vague about what was going on.

Q5 Another popular question, but scoring a relatively low average mark overall; however, this was partly due to the high number of partial attempts, and good efforts usually scored around 14 marks. Surprisingly, the greatest difficulty was found in the differentiation in part (ii). There were lots of marks for the curve-sketching, and several easy features to work with: principally the symmetry and the asymptotes (and the behaviour of the functions on either side of these). For many who struggled, the biggest problem lay in where to put the *y*-axis, which was largely immaterial. As mentioned already, differentiation attempts were rather poor on the whole, with muddling of the *Chain, Product* and *Quotient Rules*. Even for those who differentiated correctly, extracting the factor (2x - a - b) proved too tough, despite the fact it might have been obvious with a bit of thought.

Q6 This was the second least popular of the pure maths questions, partly (it seems) because many candidates did not know what was meant by the term *cyclic quadrilateral*. The other immediate hurdle was that candidates needed to know that "*opposite angles of a cyclic quad. are supplementary*". We know this because of the large number of "attempts" that got no further than an initial diagram and a bit of working. Thus, the question was even less popular than the raw figures show. In reality, the question involved little more than some GCSE-level trigonometry, the *difference of two squares factorisation* and the result $\sin^2 + \cos^2 = 1$. Those who overcame the initial hurdles scored highly.

Q7 This question was the least popular of the pure maths questions by a considerable margin, and attempts at it were usually fairly poor. In fact, very few candidates got beyond the opening (given) result. The barrier to further progress was almost invariably the failure to realise that all points on a circle centre O and radius 1 have position vectors that satisfy $\mathbf{x} \cdot \mathbf{x} = 1$.

Q8 Well over a half of all candidates attempted this question but, on average, it proved to be the least well scoring. The initial inequality was usually well handled, but most of the remaining parts of the question were poorly handled in very circuitous ways, with few candidates being very clear in either what they were trying to prove or how. The first, given, result simply follows from equating for q^2 in two successive cases of the given recurrence definition. This result was then supposed to help with the following result, but almost no-one seemed to realise this, and attempts at inductive proofs were quite common at this stage (usually unsuccessfully). Candidates' confidence had clearly ebbed away well before the final part of the question, and so there were very few attempts at the two cases of the final paragraph.

Q9 Despite its obviously algebraic nature, and incorporating inequalities, this was a remarkably popular question with candidates, more than 400 of whom chose to do it. Moreover, it also proved to be the most successful question on the paper, with the average mark exceeding a score of 12. Marks that were lost generally arose from a lack of care with signs (directions) or a failure to justify the direction of the inequality from the physical nature of the situation.

Q10 This was the least popular question on the paper, attracting the poorest efforts and having the weakest mean score (under 4 marks). Around half of attempts foundered at the very outset by failing to have "the vertical plane containing the rod ... perpendicular to the axis of the cylinder". Those candidates who resolved horizontally and vertically, instead of parallel and perpendicular to the rod, invariably ended up with a terrible mess that they simply couldn't sort; a few forget to take moments at all and were thus unable to make much progress towards the answers required.

Q11 There were many very capable attempts at this question, taken by around a quarter of all candidates. At some stage, a general approach was required to the use of the principle of conservation of linear momentum; some resorted to "pattern-spotting" (which lost them a couple of marks) and others to an inductive approach, collision by collision, which worked well though was generally a lengthier bit of work. Some candidates mixed up *n* with *N* in the following part, while others incorrectly considered > rather than \geq and ended up missing the answer by 1. A little bit of care was needed with the summation in the final part, and there was a bit of fiddling going on in order to get the given answer. Nevertheless, it was pleasing to see the principles understood well, even if the details were less carefully attended to.

Q12 This was not a popular question, and most attempts petered out after part (i), which was usually handled very well, even when the situation was split into more cases than was strictly necessary. Indeed, few made much of a serious attempt at (ii), mainly because they were either finding the range of *p* such that (i)'s given answer was equal to $\frac{3}{7}$, or solving 2.5 < E(X) < 3.5, where *X* was the number of days that the light was on.

Q13 This question drew very little interest from candidates. Most attempts gained the first couple of answers and then differentiated to find the pdf of *Y*. In the attempts to find E(Y) and $E(Y^2)$, most candidates rightly attempted *integration by parts*, although some coefficients went astray when either making a substitution or comparing the integrals with the corresponding ones of the standard normal distribution. Slightly surprisingly, it was relatively common to find $E(Y^2)$ correct but the variance incorrect, as candidates failed to make this modest extra step without error.

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